Printed Pages : 4 **EAS-203**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610 **Roll No.** | | | | | | | | | | |

B. Tech.

(Semester-II) Even Semester Theory Examination, 2012-13

MATHEMATICS-II

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as per instructions.

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. 2×10=20

1. (a) Find the integrating factor for the differential equation $\frac{dy}{dx}+\frac{3}{x}y=\frac{1}{x^4}$.

(b) Find the value of the integral $\int_{-1}^{+1} P_n^2(x).dx$ for $n=1$.

(c) Find the Laplace transform of $\sin^2(t)$.

(d) Calculate the value of a_0 in Fourier expansion of $f(x)=e^{-x}$ in the interval $(0,\ 2\pi)$.

(e) Find the particular integral of $(D^2+a^2)y=\sin ax$.

(f) Find the solution of $\frac{\partial^3 z}{\partial x^3}=0$.

(g) Find the solution of heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$.

(h) Evaluate :

$$\int_0^3 x^2\sqrt{(3-x)}.$$

(i) Find $L\left\{\frac{F(t)}{t}\right\}$ transformation

(j) Find the value of $P_{2n}(0)$.

Section-B

Attempt any *three* parts of this question. Each part carries 10 marks. 10×3=30

2. (a) The deflection of a strut of length L with one end ($x = 0$) built in and the other supported and subjected to end thrust P, satisfies the differential equation $\frac{d^2y}{dx^2}+a^2y=\frac{a^2R}{P}(L-x)$. Prove that deflection curve is :

$$y=\frac{R}{P}\left(\frac{\sin ax}{a}-L\cos ax+L-x\right),$$

where $aL = \tan aL$.

(b) Solve in series the differential equation :

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+xy=0.$$

(c) Using Laplace transform, solve the differential equation:

$$\frac{d^2x}{dt^2}+9x=\cos 2t, \text{ if } x(0)=1, x\left(\frac{\pi}{2}\right)=-1.$$

(d) Find the Fourier series representation of the function $f(x) = x - x^2$ in the interval $(-\pi, \pi)$ and hence show that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots=\frac{\pi^2}{12}$

(e) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $y = y_0 \sin^3\frac{\pi x}{L}$, if it is released from rest from their position, find the displacement $y(x, t)$.

Section-C

Attempt any *two* parts of each question from this Section. Each question carries 10 marks. 5×2×5=50

3. (a) Solve the following differential equation :

$$(D^2-4D+4)y=8x^2e^{2x}\sin 2x.$$

(b) By Changing the independent variable solve :

$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log(1+x).$$

(c) Apply method of variation of parameters to solve the ordinary differential equation $\frac{d^2y}{dx^2}+y=\tan x.$

4. (a) Express the function $f(x)=x^4+3x^3-x^2+5x-2$ in terms of Legendre's polynomials.

(b) Prove that :

$$\int xJ_0^2(x)dx=\frac{1}{2}x^2[J_0^2(x)+J_1^2(x)]+c$$

(c) If α and β are the roots of $J_n(x)=0$, then prove that :

$$\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx=0,$$

when $\alpha\neq\beta$.

5. (a) Using Laplace transform evaluate the integral :

$$\int_0^\infty \frac{e^{-t}\sin^2 t}{t}dt.$$

(b) A periodic square wave function $F(t)$ in terms of unit step function, is written as $F(t)=k[u_0(t)-2u_a(t)+2u_{2a}(t)-2u_{3a}(t)+\ldots\ldots]$ find its Laplace transform.

(c) State convolution theorem and hence evaluate :

$$L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}.$$

6. (a) Solve the linear partial differential equation $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$; where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

(b) Solve $\frac{\partial^3 z}{\partial x^3} - \frac{\partial^3 z}{\partial y^3} = x^3 y^3$.

(c) Find the Fourier series as far as the second harmonic to represent the function given by the following table :

x	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	360°
$f(x)$	2.34	3.01	3.69	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

7. (a) Solve the method of separation of variables :

$$4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u;\ u = 3e^{-x} - e^{-5x} \text{ when } t = 0.$$

(b) Solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} = 0$; using the transformation $v = x + y$, $z = 2x - y$.

(c) A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature function $u(x, t)$.